The Relaxation of Heavy Ion Impurities in an Anisotropic Deuterium Plasma

by

G. Lehner and F. Pohl

IPP 1/91 November 1968

INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

INSTITUT FÜR PLASMAPHYSIK

GARCHING BEI MÜNCHEN

The Relaxation of Heavy Ion Impurities
in an Anisotropic Deuterium Plasma

by

G. Lehner and F. Pohl

IPP 1/91 November 1968

Die nachstehende Arbeit wurde im Rahmen des Vertrages zwischen dem Institut für Plasmaphysik GmbH und der Europäischen Atomgemeinschaft über die Zusammenarbeit auf dem Gebiete der Plasmaphysik durchgeführt.

IPP 1/91	G. Lehner F. Pohl	The Relaxation of Heavy Ion Impurities in an Anisotropic Deuterium Plasma November 1968

Abstract

The relaxation of the anisotropy of impurity ions in an anisotropic plasma is discussed. The treatment is based on the assumption that all velocity distribution functions are elliptic initially and remain elliptic during the relaxation process. The problem thus reduces to a set of ordinary differential equations for the perpendicular and parallel temperatures of the different kinds of plasma particles. One may hope that such an approximate description of the relaxation is of some help in the interpretation of experimental results, as obtained by, for instance, Bogen et al [4].

Some examples are treated numerically.

1) Introduction

The plasmas produced by fast theta-pinches, for instance, are strongly anisotropic. Possibilities of directly measuring the plasma anisotropy (more precisely: the anisotropy of the deuterons; the electrons can be assumed to be isotropic in good approximation) by making use of the neutrons emitted from the pinched plasma have been discussed both theoretically [1] and experimentally [2.3] . Bogen, Rusbüldt, and Schlüter [4] have recently used another method. The plasma contains some high-Z impurities or, if it should be very pure, a controlled amount of such impurities may be added. One can then measure the Doppler width of their spectral lines as seen from different directions, say, parallel and perpendicular to the axis of the magnetic field. Bogen, Rusbüldt, and Schlüter have done this for CV, NVI, and OVII lines. One then faces the problem of finding how the anisotropies of different kinds of particles in the plasma are connected with one another, i.e. how the anisotropies relax owing to collisions between all kinds of particles existing in the plasma. A precise treatment of the problem would require a numerical solution of the kinetic equation of the plasma, which in turn has to be based on a sufficiently precise knowledge of the initial conditions that have to be obtained from the experiment. This information is not available, however. It thus seems more promising to make simplifying assumptions which are adapted to the experimental knowledge obtainable by present "diagnostic" techniques. These yield, for instance, some mean values of the particle energies (these particles being the deuterons in the case of neutron measurements, the heavy ions in the case of spectroscopic measurements) for the direction of observation. The resolution is not sufficient, however, for supplying detailed information on the velocity distribution itself. In view of this situation, it is useless to calculate more than the time behaviour of mean energies. The most natural assumption then is to take initially elliptic velocity distributions which are supposed to remain so during the relaxation process. In this approximation each kind of particles is described by two "temperatures", a perpendicular one and a parallel one (with reference to the direction of the magnetic field). This is due to the tacit assumption of rotational symmetry (in the general case one would have three "temperatures"). The assumption of elliptic distributions is also very natural in the sense that it allows the description of the transition to thermodynamic equilibrium, which is a basic requirement for the choice of the functional form of the velocity distribution. The problem has been discussed on these assumptions in previous papers, first with the help of the Fokker-Planck equation [5] and then with the help of the Balescu-Lenard equation [6]. The latter treatment shows that (except for extreme parameters) the inclusion of collective effects (by taking the Balescu-Lenard equation) is not necessary. This had to be expected and thus the situation is the same as in the small anisotropy case, which has been treated by Wu et al [7].

2) Equations

In references [5,6] the problem has been discussed for singly charged ions and electrons. By introducing the ionic charge Z the results can easily be generalized. We use equations (34) to (39) of reference [6], which (taking into account m kinds of particles denoted by subscripts i, k running from 1 to m) may be written as follows:

$$\frac{dT_i}{dt} = \sum_{k=1}^{m} J_{ik}\, n_k \tag{1}$$

$$\frac{d(T_{\perp i} - T_{\parallel i})}{dt} = \sum_{k=1}^{m} K_{ik}\, n_k \tag{2}$$

where the collision terms are given by

$$J_{ik} = \frac{8e^4 Z_k^2 Z_i^2 \ln\Lambda}{3\left[m_k(T_{\perp i} - T_{\parallel i}) + m_i(T_{\perp k} - T_{\parallel k})\right]} \left(\frac{2\pi m_i m_k}{m_i T_{\parallel k} + m_k T_{\parallel i}}\right)^{1/2} \cdot \left\{ \frac{(m_i + m_k)(T_{\perp i} T_{\parallel k} - T_{\perp k} T_{\parallel i})}{m_i T_{\perp k} + m_k T_{\perp i}} + \left[(T_{\perp k} - T_{\parallel k}) - (T_{\perp i} - T_{\parallel i})\right] \varphi(x_{ik}) \right\} \tag{3}$$

and

$$K_{ik} = \frac{2e^4 Z_k^2 Z_i^2 \ln\Lambda\,(2\pi)^{1/2}}{(m_i T_{\parallel k} + m_k T_{\parallel i})^{3/2}(m_i m_k)^{1/2}\, x_{ik}(1+x_{ik})} \cdot$$

$$\cdot\left\{(3M+L) + N x_{ik} - \varphi(x_{ik})\left[(3M+L) + (N+M+L)x_{ik} + (N-2M)x_{ik}^2\right]\right\} \qquad (4)$$

For i = k, i.e. for collisions among particles of one kind,

$$K_{ii} = \frac{6e^4 Z_i^4 \ln\Lambda}{T_{\perp i} - T_{\parallel i}}\left(\frac{\pi T_{\parallel i}}{m_i}\right)^{1/2}\left\{3 - \varphi(x_{ii})(3 + x_{ii})\right\} \qquad (5)$$

which corresponds to the result obtained by Kogan [8]. We still have to define L, M, N, ,x_{ik}:

$$L = 2(m_i^2 + m_i m_k)(T_{\parallel k} - T_{\perp k}) \qquad (6)$$

$$M = m_i m_k T_{\parallel k} + 2 m_i m_k T_{\parallel i} + 3 m_k^2 T_{\parallel i} \qquad (7)$$

$$N = 5 m_i m_k T_{\parallel k} + 6 m_i m_k T_{\parallel i} + 9 m_k^2 T_{\parallel i} + 2 m_i^2 T_{\parallel k} \qquad (8)$$

$$\varphi(x) = \frac{\operatorname{arctg}\sqrt{x}}{\sqrt{x}} = 1 - \frac{x}{3} + \frac{x^2}{5} - \dots \qquad (9)$$

$$x_{ik} = \frac{m_i(T_{\perp k} - T_{\parallel k}) + m_k(T_{\perp i} - T_{\parallel i})}{m_i T_{\parallel k} + m_k T_{\parallel i}} \qquad (10)$$

$\overline{T}_i$ is the mean temperature (proportional to the energy) of kind i particles:

$$\overline{T}_i = \frac{2T_{\perp i} + T_{\parallel i}}{3} \qquad (11)$$

In writing down equations (1) and (2) we have chosen $\overline{T}_i$ and $T_{\perp i} - T_{\parallel i}$ as our variables, (instead of $T_{\perp i}$ and $T_{\parallel i}$). Equation (1) thus describes the energy exchange between different kinds of particles, while equation (2) describes the relaxation of anisotropy. Let us note that

$$J_{ik} = - J_{ki} \qquad (12)$$

i.e. the J_{ik} are antisymmetric with respect to their indices, this being a consequence of energy conservation. On the other hand, there is no such relation for the K_{ik} :

$$K_{ik} \mp \pm K_{ki} \tag{13}$$

The other quantities used in the equations are:

n_i particle density
$T_{\perp i}$ perpendicular temperature
$T_{\parallel i}$ parallel temperature
Z_i charge of particles (in units of e)
e elementary charge
m_i particle mass
$\ln\Lambda$ Coulomb logarithm

3) Discussion of the equations

In certain cases the equations may be simplified. The time scales for different kinds of particles depend on their masses. Due to their small mass electrons relax faster than ions do. Thus the electrons may be considered as isotropic at least as far as ionic time scales are concerned. If the index e denotes electrons and i same kind of ions, we have in this case

$$x_{ie} = \frac{m_i(T_{\perp e} - T_{\parallel e}) + m_e(T_{\perp i} - T_{\parallel i})}{m_i T_{\parallel e} + m_e T_{\parallel i}} \ll 1 \tag{14}$$

$$m_i T_e \gg m_e T_{\perp,\parallel i} \tag{15}$$

i.e. if the ratio of ion temperature and electron temperature is not extremely large. So one may expand φ, equation (9), and obtain linearised results even for large ion anisotropy:

$$J_{ei} = -J_{ie} \approx \frac{8e^4 Z_i^2 \ln\Lambda}{3m_i}\left(\frac{2\pi m_e}{T_e^3}\right)^{1/2}(T_i - T_e) \tag{16}$$

$$K_{ei} \approx 0 \tag{17}$$

$$K_{ie} \approx -\frac{8e^4 Z_i^2 \ln\Lambda}{3m_i}\left(\frac{2\pi m_e}{T_e^3}\right)^{1/2}\left(T_{\perp i}-T_{\parallel i}\right) \tag{18}$$

This is a consistent set of equations. To avoid confusion let us, however, add the following remark. If one kind of particles is isotropic while the other kinds are not, the anisotropic kinds produce an anisotropy of the initially isotropic kind also. So even for isotropic electrons $K_{ei} \neq 0$ if the ions are anisotropic. The point is that K_{ei} is of the order $\frac{me}{mi}$ and can be neglected in the above set of equations. Let us consider a plasma composed of electrons and one kind of ions only. It can then be shown that an anisotropy $T_{\perp i}-T_{\parallel i}$ of the ions produces the following anisotropy of the electrons:

$$T_{\perp e}-T_{\parallel e} = \alpha\,\frac{m_e}{m_i}\left(T_{\perp i}-T_{\parallel i}\right) \tag{19}$$

where

$$\alpha = \frac{4Z_i^2\frac{n_i}{n_e}}{3\left(\sqrt{2}+2Z_i^2\frac{n_i}{n_e}\right)} = \frac{4Z_i}{3\left(\sqrt{2}+2Z_i\right)} \tag{20}$$

Then

$$X_{ie} = (1+\alpha)\,\frac{m_e\left(T_{\perp i}-T_{\parallel i}\right)}{m_i T_{\parallel e}+m_e T_{\parallel i}} \approx (1+\alpha)\,\frac{m_e\left(T_{\perp i}-T_{\parallel i}\right)}{m_i T_{\parallel e}} \ll 1 \tag{21}$$

On expanding one now obtains equations (16) and (18) as given above. The factor α cancels and does not appear in J_{ei} and K_{ie}. It does, however, show up in the expression for K_{ei}:

$$K_{ei} \approx \frac{16e^4 Z_i^2 \ln\Lambda}{15m_i}\left(\frac{2\pi m_e}{T_e^3}\right)^{1/2}\left(1-\frac{3}{2}\alpha\right)\left(T_{\perp i}-T_{\parallel i}\right) \tag{22}$$

We can linearise equation (5) for electrons if they are nearly isotropic also:

$$K_{ee} \approx -\frac{8e^4 \ln\Lambda}{5}\left(\frac{\pi}{m_e T_e^3}\right)^{1/2}\left(T_{\perp e}-T_{\parallel e}\right) \tag{23}$$

Therefore

$$\frac{d(T_{\perp e}-T_{\parallel e})}{dt} = n_i K_{ei} + n_e K_{ee} = 0 \tag{24}$$

if

$$T_{\perp e}-T_{\parallel e} = \frac{\alpha m_e}{m_i}\left(T_{\perp i}-T_{\parallel i}\right) = \frac{n_i}{n_e}\cdot\frac{2\sqrt{2}}{3} Z_i^2 \frac{m_e}{m_i}\left(1-\frac{3}{2}\alpha\right)\left(T_{\perp i}-T_{\parallel i}\right) \tag{25}$$

This shows that the Ansatz (19) is justified and gives equation (20) for α. We see that the electrons are slightly anisotropic as long as the ions are. Because $K_{ei} \sim \frac{m_e}{m_i} K_{ee}$ this anisotropy is negligibly small, however.

If the electrons interact with several kinds of particles, the equations just given can easily be generalized:

$$T_{\perp e}-T_{\parallel e} = \sum_{i \neq e} \alpha_i \frac{m_e}{m_i}\left(T_{\perp i}-T_{\parallel i}\right) \tag{26}$$

where

$$\alpha_i = \frac{4\,\frac{n_i}{n_e} Z_i^2}{3\left(\sqrt{2}+2\sum\limits_{k \neq e} \frac{n_k}{n_e} Z_k^2 \frac{m_k}{m_i}\right)} \tag{27}$$

Our treatment does not include magnetic field effects, but these may become important in the form of micro-instabilities, for example. In applying the Balescu-Lenard equation one supposes that the velocity distributions referred to are microstable. If this is not the case(and the magnetic field may in many cases produce microinstabilities of various types such as, for instance, mirror instabilities, firehose instabilities, loss cone instablilities etc.) the Balescu-Lenard equation breaks down and neglecting collective effects, i.e. the Fokker-Planck treatment of the problem, is no longer justified.

If no instability is involved the magnetic field effects can be included. The heating produced by a magnetic field varying with time can be described by an adiabatic heating term

$$\frac{dT_{\perp i}}{dt} = \frac{T_{\perp i}}{B} \frac{dB}{dt} \qquad (28)$$

as has been done in previous applications [3,4]. We are assuming throughout that the plasma is homogeneous and also imbedded in a homogeneous magnetic field. This means that our equations refer to suitably chosen spatial mean values.

For very high magnetic fields it may become necessary to use in the Coulomb logarithm the mean values of Larmor radii which replace the Debye length if they become smaller than the Debye length. Since only the logarithm is concerned, however, this is not a very important correction.

To describe the problem completely, we should have to include more equations describing the processes of ionization and recombination and the energy transfer between different kinds of ions due to these processes. Consider for instance, a plasma with deuterons, electrons, and carbon impurities. One may then have several heavy ions at the same time, say, CIV, CV, CVI. Owing to recombination and ionization processes a given particle does not always belong to the same kind of ions and it carries its energy from kind to kind. In principle, this processes can be taken into account. This will not be done in the present report, however.

To compute the relaxation one needs in principle at least the densities of all kinds of particles. Experimentally, the densities of impurity ions are not very well known, or not known at all. It is not necessary to know these densities if they are sufficiently small. In this case the relaxation is due to collisions with electrons and deuterons while heavy ion - heavy ion collisions can be neglected, and so the heavy ion densities do not enter because corresponding terms in equations (1) and (2) are small. One has to be careful, however, because owing to the factor Z^2 these terms may become important even for relatively low impurity concentrations.

4) Numerical examples

Let us now give some numerical solutions of the equations discussed.

In Figure 1 we consider a plasma containing electrons, deuterons, and CV-ions. Initial temperatures are $T_{\perp o} = T_{\perp do} = T_{\perp C\overline{V}o} = 900\,eV = T_{\perp eo}$ and $T_{\parallel o} = T_{\parallel do} = T_{\parallel eo} = T_{\parallel C\overline{V}o} = 100\,eV$.

The particle densities of electrons and deuterons are equal, $n_e = n_d = 10^{17}$, while $n_{C\overline{V}} \Rightarrow 0$.

The electrons relax very fast, i.e. within a time during which deuterons and CV-ions are essentially unaffected. If, for comparison, the electrons are treated as isotropic from the beginning, the behaviour of deuterons and CV-ions is not changed. The anisotropy of the ions relaxes somewhat faster than that of the deuterons. This is due to the factor $Z^2_{C\overline{V}} = 16$ which overcompensates the factor 6 in the masses. A change in the particle density changes the time scale by the same factor. The Coulomb logarithm has been replaced by 10 in all cases.

Figures 2 to 4 describe the behaviour of a similar plasma. The difference is that the electrons are isotropic and their initial temperature is 100 eV. The CV-ions are anisotropic (initial perpendicular and parallel temperatures are 900 eV and 100 eV resp.). The deuterons are also anisotropic. Their initial parallel temperature is 100 eV, while the initial perpendicular temperature takes different values, viz. 300, 600, and 900 eV. Particle densities are as before in the Figure 1 case.

For reasons which will become clear later the computation starts at 0.2 µsec as initial time. Figures 2 to 4 show that the CV-ions relax faster than the deuterons. Even if the initial anisotropy of the CV-ions is larger than that of the deuterons initially it becomes smaller after a short time.

Figures 5 to 7 refer to the same parameters as Figures 2 to 4, the only difference being the addition of an adiabatic heating term (magnetic field). The data are chosen so as to correspond to the case of reference [4], except for the initial temperature of the deuterons, which is not given in reference [4] since it could not be measured. We have again, as above, considered 1oo eV parallel and 3oo, 6oo, 9oo eV perpendicular temperature. The reason for this choice is as follows. The theta-pinch compression may roughly be divided into two stages. The first stage consists of a rapid radial contraction in which the heating of the plasma is mainly due to a shock wave. The second stage may be described as adiabatic heating due to the increasing magnetic field. Thus the initial conditions for the adiabatic heating are determined by the preceding shock wave. The shock wave mainly heats particles with higher mass. If there were no competing processes the perpendicular temperatures reached would be proportional to the masses. This, however, is not to be expected. The electrons are mainly heated by collisions (Joule heating). Furthermore, the two stages, the dynamic one and the adiabatic one, overlap in a complicated way. In any case one would expect, however, that at some early time the CV ions should be essentially hotter than the deuterons (we are referring to perpendicular temperatures only). Thus we considered the perpendicular temperature of the deuterons as a parameter which we varied as mentioned above.
On the other hand, the computations show that the CV-ions relax so fast that they cool down below the deuterons within a short time. The measurements of Bogen et al [4] start at o.2 µsec. Our computation starts at the same time taking their data as initial values. One could imagine that due to the relaxation at earlier times $T_{\perp cV}$ is already smaller than $T_{\perp d}$ at o.2 µsec. Such an assumption would not fit with the measurements, however. An initial value $T_{d\perp}$ (o.2 µsec) = 9oo eV gives $T_{d\perp}$ (o.7 µsec) $\approx$ 1.4 keV, a value much too high in comparison with the measured value of about 5oo eV (from the neutron output). This latter value is compatible only with a relatively small initial deuteron temperature of about 4oo eV. The magnetic field is also plotted

for comparison. If there were no relaxation all perpendicular temperatures would follow the magnetic field.

5) Conclusion

The measurement of the anisotropy of a plasma via the anisotropy of impurities as introduced by Bogen et al. [4] can yield very valuable information. In using this method one should be very careful with the interpretation, however. There is no justification, at least in general, in identifying the anisotropy of the impurities with that of the deuterons. Different kinds of particles relax in a different way. So in drawing conclusions the measurements should be supported by computations. The theory given in the present report is still oversimplified, but we may hope that it describes the relaxation sufficiently well to be of some help in the interpretation of experimental results.

References

[1] G. Lehner and F. Pohl, Bericht IPP 1/60 (1967) and Z.Phys. 207, (83) 1967 and also Los Alamos Report LA-TR-67-113 (English translation)

[2] C. Andelfinger et al., Bericht IPP 1/67 (1967)

[3] C. Andelfinger et al., Proc. of Conf. on Pulsed High Density Plasmas, Report LA-3770 (1967) Paper G 2.

[4] P. Bogen et al., Conference on Plasma Physics and Controlled Nuclear Fusion Research, Novosibirsk, USSR, August 1968, Paper CN-24/K-10

[5] G. Lehner, Z. Phys. 206 (284) 1967

[6] G. Lehner and F. Pohl, in press, Z. Phys.

[7] C.S. Wu et al., Phys. Fluids 8 (1126) 1965

[8] V.J. Kogan, Plasma Physics and the Problem of Controlled Thermonuclear Reactions, Vol. I p. 153, N.Y. Pergamon Press 1961

Figures

Figure 1 Comparison of the relaxation rates for electrons, deuterons, and CV-ions. $n_e = n_d = 10^{17}$; $n_{CV} = 0$

Figure 2 Relaxation of deuterons and CV-ions.
Electrons isotropic. $n_e = n_d = 10^{17}$; $n_{CV} = 0$
Initial conditions at $t = 0{,}2$ µsec:
$T_{\perp CVo} = 900$ eV ; $T_{\parallel CVo} = 100$ eV; $T_{eo} = T_{\perp eo} = T_{\parallel eo} =$ 100 eV

$T_{\parallel do} = 100$ eV ; $T_{\perp do} = 300$ eV

Figure 3 Same as Figure 2 with $T_{\perp do} = 600$ eV

Figure 4 Same as Figure 2 with $T_{\perp do} = 900$ EV

Figure 5 Same as Figure 2, but including adiabatic heating due to a magnetic field ($B \sim \sin \frac{2\pi t}{\tau}$; $\tau/4 = 0{,}7$ µsec)

Figure 6 Same as Figure 5 with $T_{\perp do} = 600$ eV

Figure 7 Same as Figure 5 with $T_{\perp do} = 900$ eV

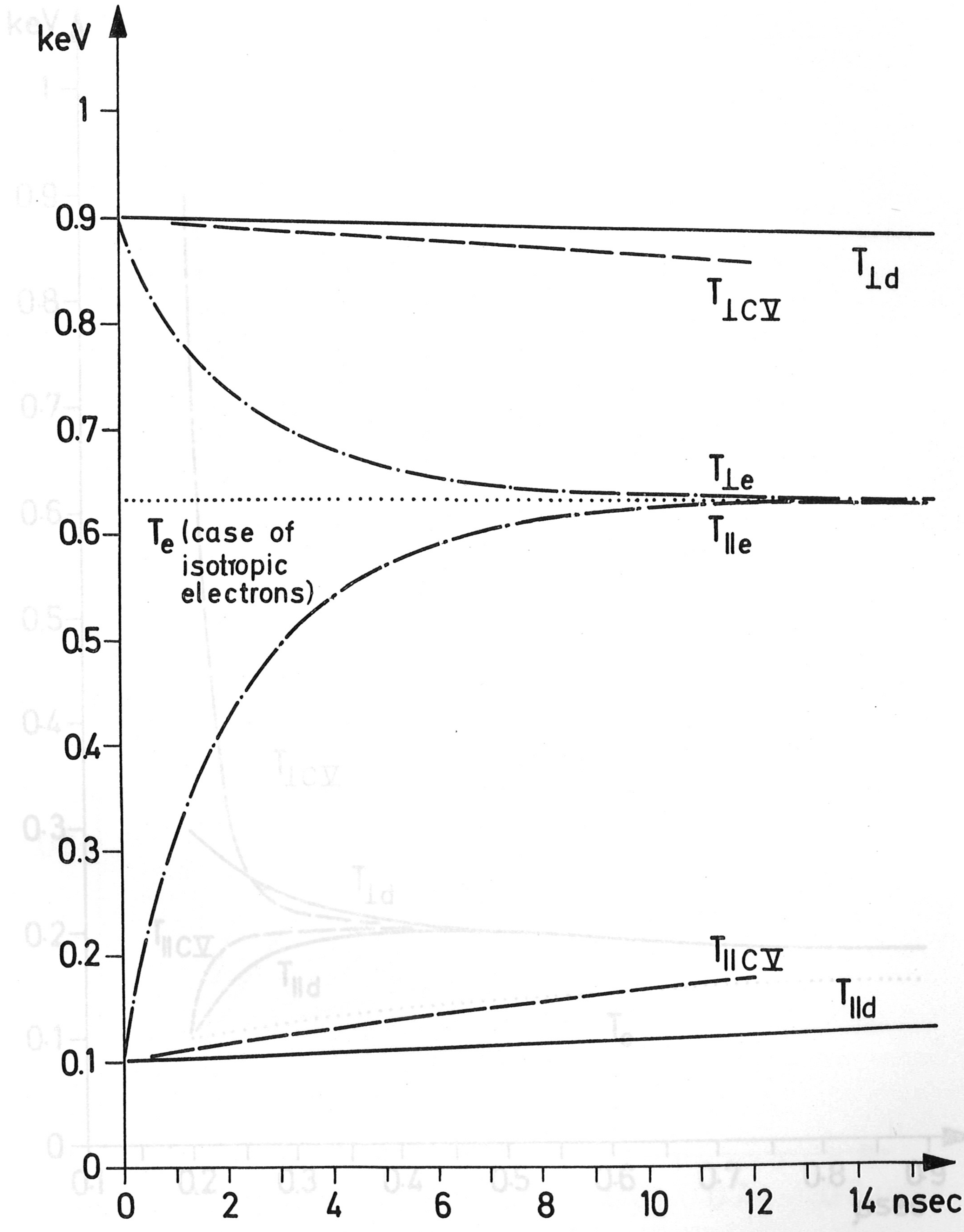
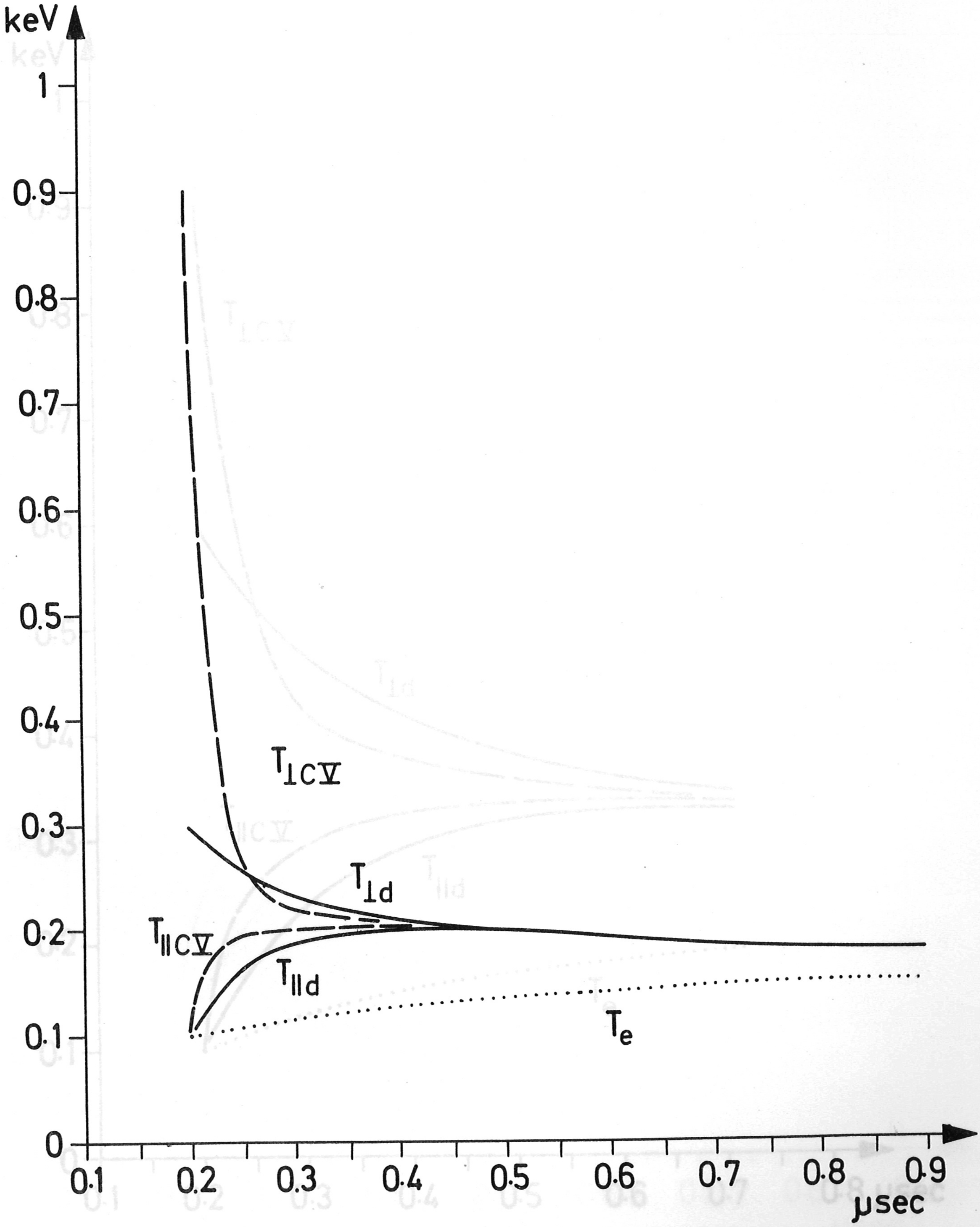

keV
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$T_{\perp d}$
$T_{\perp C V}$
$T_{\perp e}$
$T_{\parallel e}$
T_e (case of isotropic electrons)
$T_{\parallel C V}$
$T_{\parallel d}$
0
2
4
6
8
10
12
14 nsec

Figure 1



keV
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
μsec
$T_{\perp CV}$
$T_{\perp d}$
$T_{\parallel CV}$
$T_{\parallel d}$
T_e

Figure 2

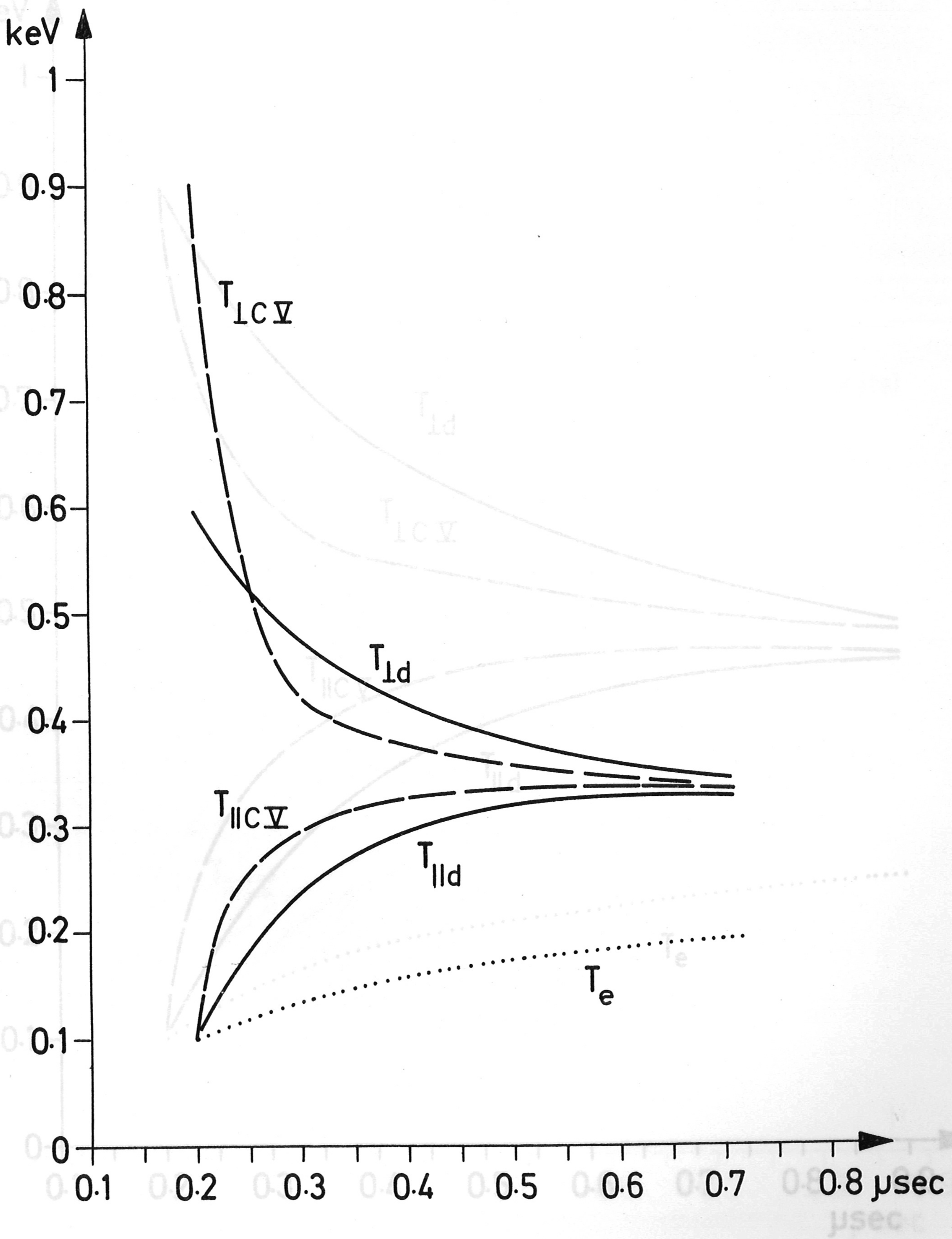

keV
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 μsec
$T_{\perp C V}$
$T_{\perp d}$
$T_{\parallel C V}$
$T_{\parallel d}$
T_e

Figure 3

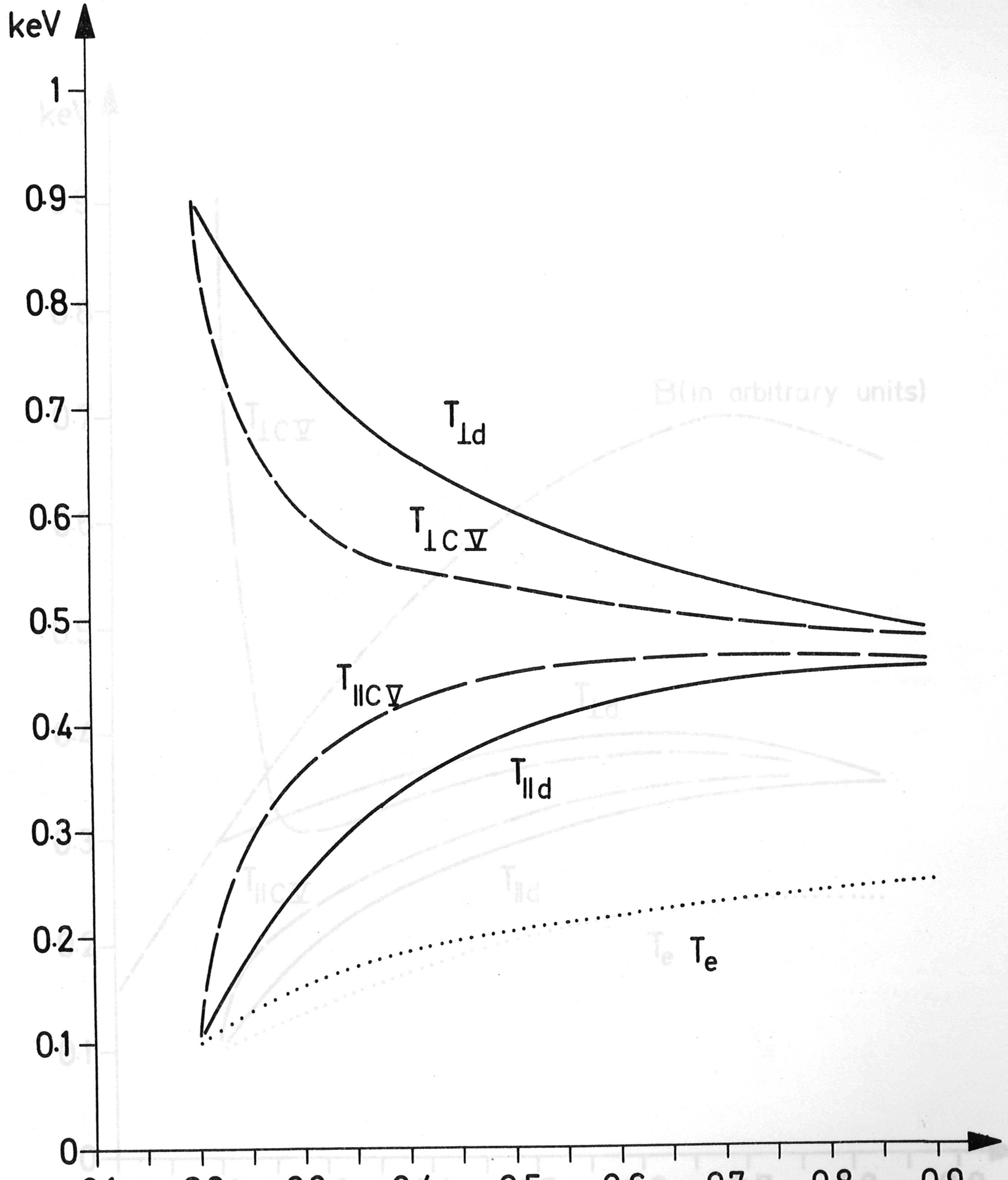

keV
1
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
$T_{\perp d}$
$T_{\perp C\,\mathrm{V}}$
$T_{\parallel C\,\mathrm{V}}$
$T_{\parallel d}$
T_e
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
µsec

Figure 4

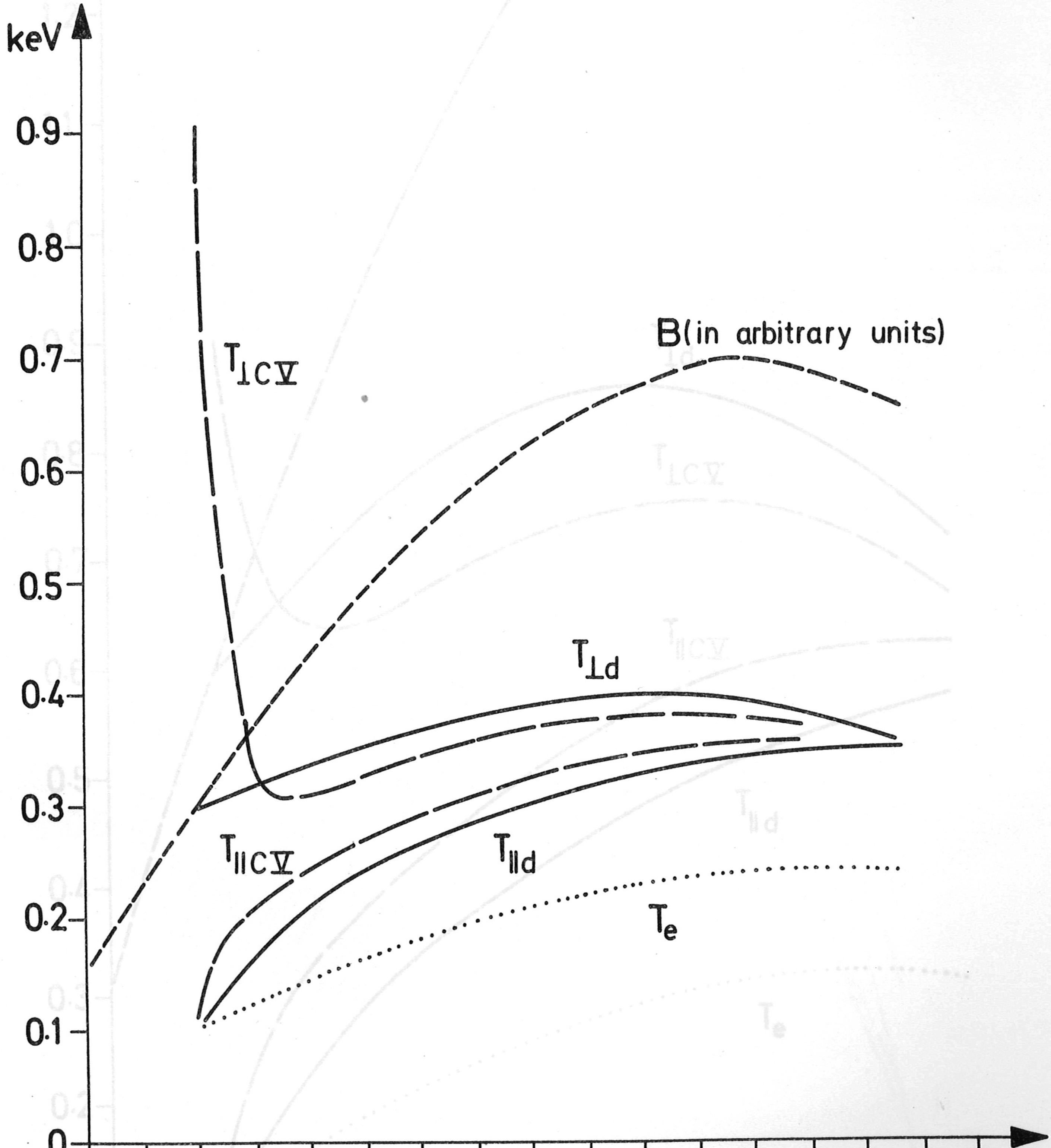
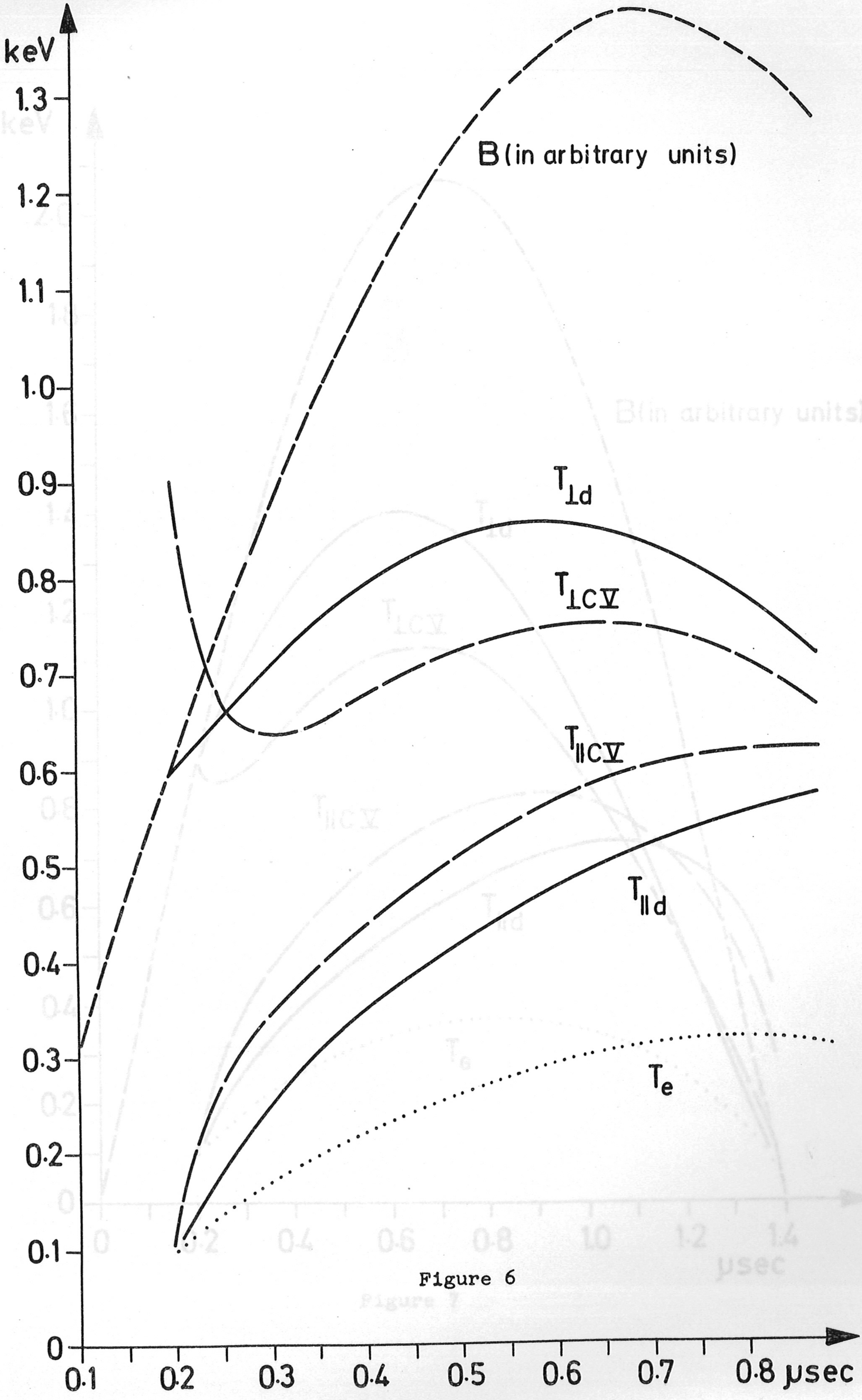

keV
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
μsec
B(in arbitrary units)
$T_{\perp CV}$
$T_{\perp d}$
$T_{\parallel CV}$
$T_{\parallel d}$
T_e

Figure 5



keV
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
B (in arbitrary units)
$T_{\perp d}$
$T_{\perp C V}$
$T_{\parallel C V}$
$T_{\parallel d}$
T_e
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8 μsec

Figure 6

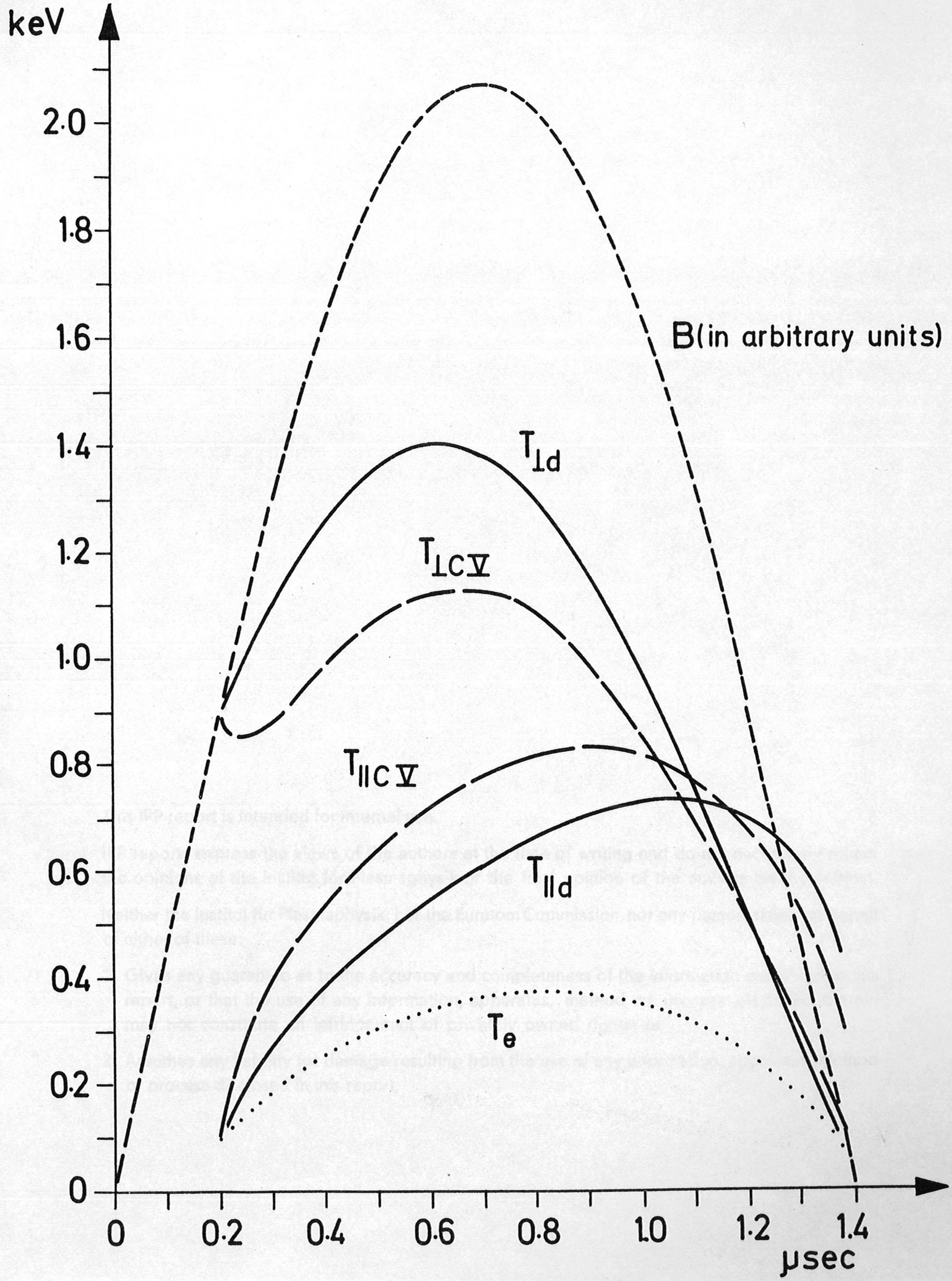

keV
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
B(in arbitrary units)
$T_{\perp d}$
$T_{\perp C V}$
$T_{\parallel C V}$
$T_{\parallel d}$
T_e
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
µsec

Figure 7